F THE AREA OF A TRIANGLE

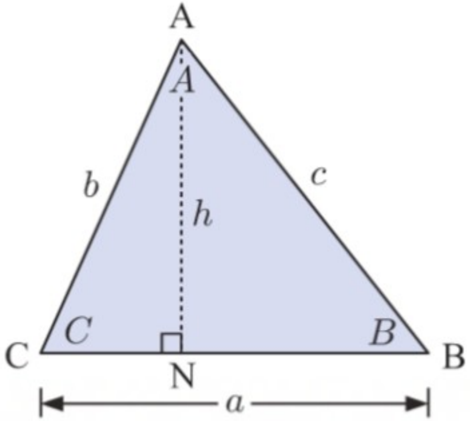
We can use trigonometry to find the area of a triangle if we are given the lengths of **two sides**, as well as the **included angle** between the sides.

Consider triangle ABC in which the sides opposite angles A, B, and C are labelled a, b, and c respectively.

Suppose N lies on [BC] such that [AN] is perpendicular to [BC].

In $\triangle$ANC, $\sin C = \dfrac{h}{b}$

$\therefore \quad h = b\sin C$



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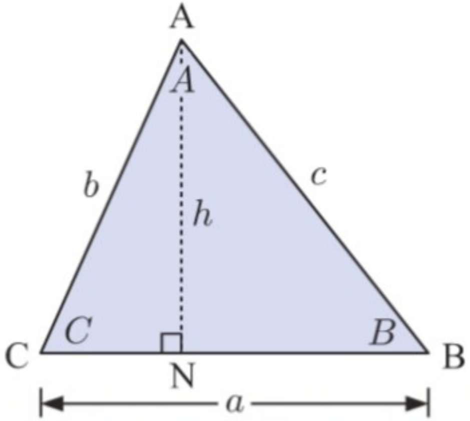

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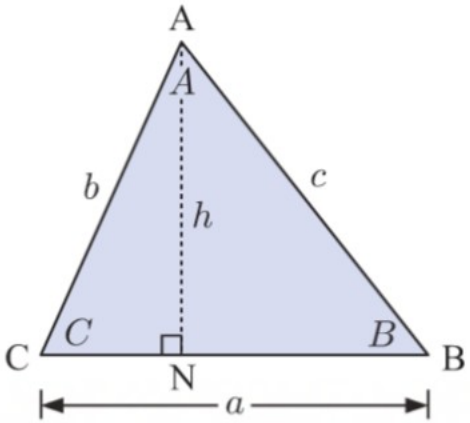

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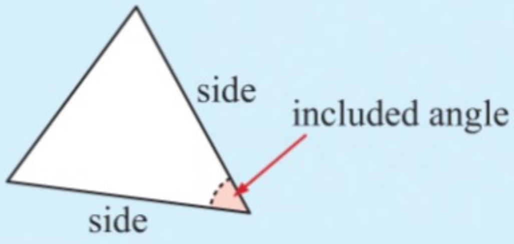
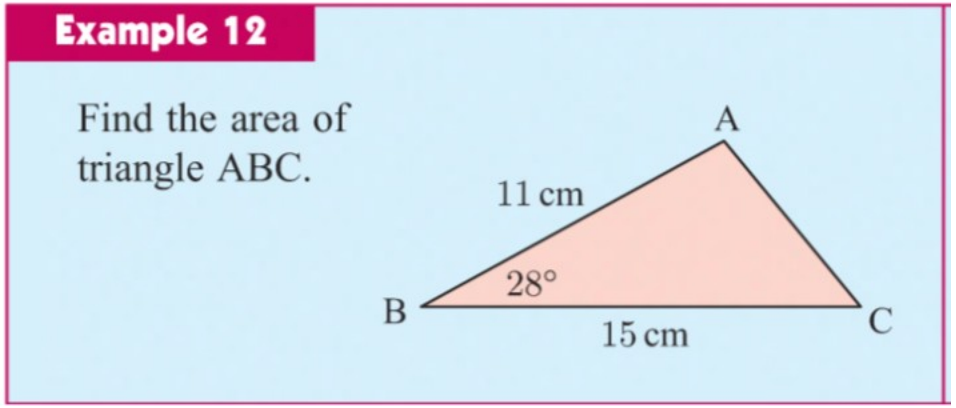
If the altitudes from B and C were drawn, we could also show that the area is $\frac{1}{2}bc\sin A$ or $\frac{1}{2}ac$ si

The **area of a triangle** is

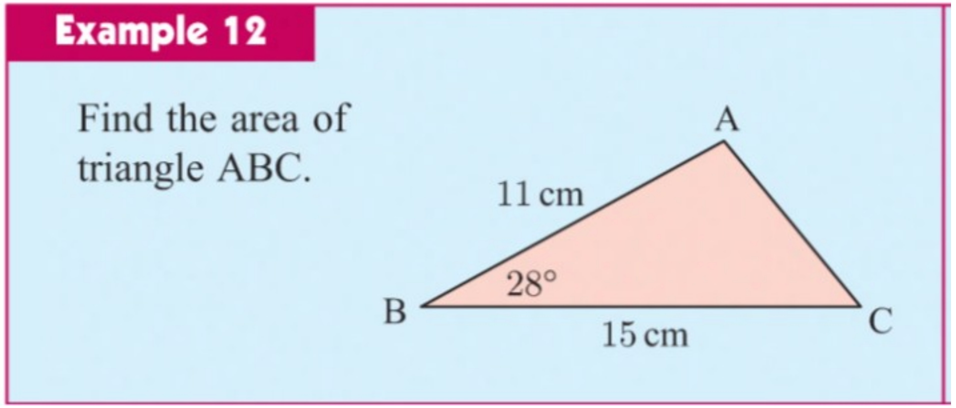
a half of the product of two sides and the sine of the included angle.





Example 12

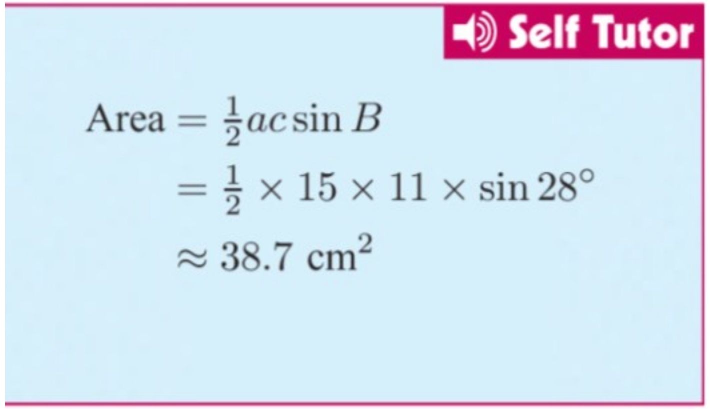
Find the area of triangle ABC.



Example 12

Find the area of triangle ABC.

A
11 cm
28°
B
15 cm
C

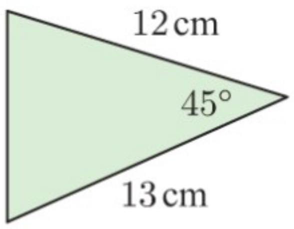


Self Tutor

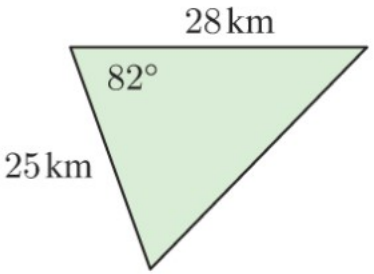
$$\begin{aligned}\text{Area} &= \tfrac{1}{2}ac\sin B\\ &= \tfrac{1}{2}\times 15\times 11\times \sin 28^\circ\\ &\approx 38.7\ \text{cm}^2\end{aligned}$$

1 Find the area of:

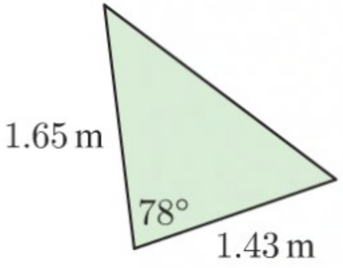
a



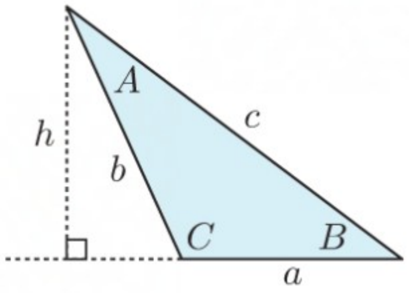
b



c

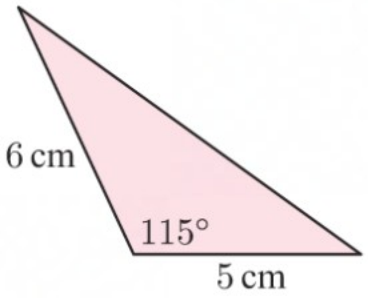


2 The proof of the area formula given above assumes that the included angle C is acute. Use the diagram alongside to prove that the formula is also true in the case where C is obtuse.
Hint: $\sin(180° - C) = \sin C$

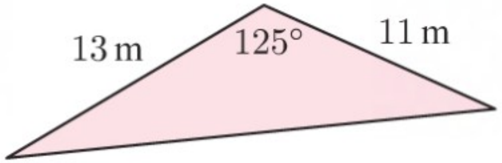


3 Find the area of:

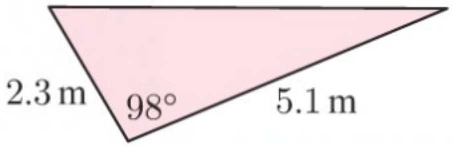
a



b

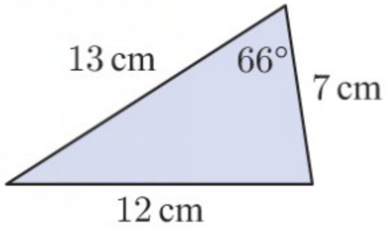


c

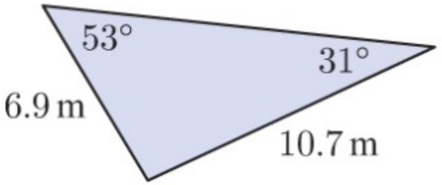


4 Find the area of:

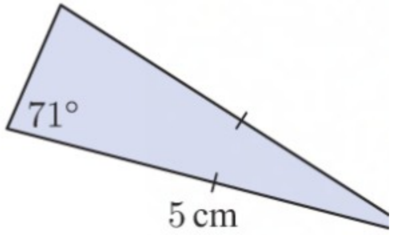
a



b

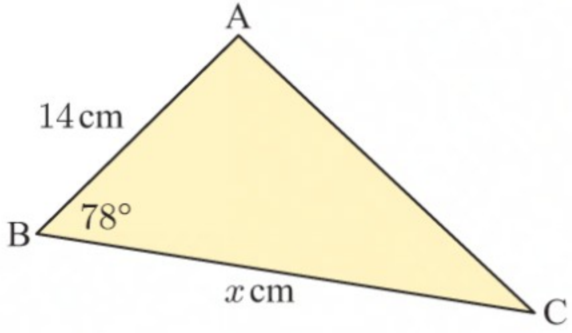


c

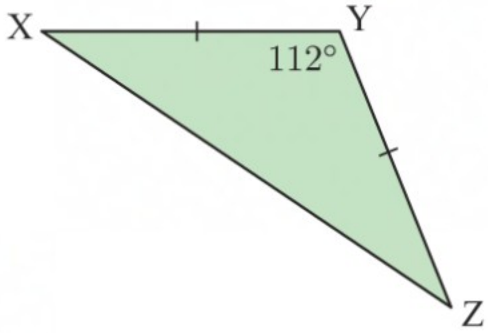


5 Find the area of a parallelogram with sides 6.4 cm and 8.7 cm, and one interior angle 64°.

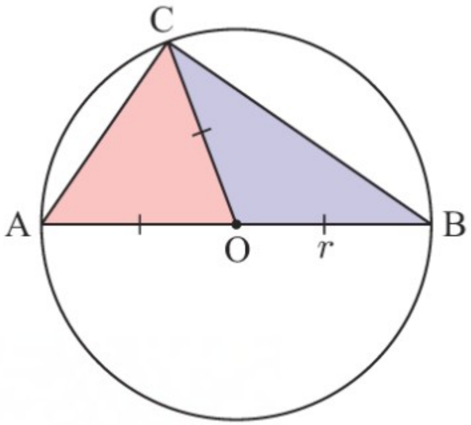
6 Triangle ABC has area 150 cm^2.
Find the value of x.



7 Triangle XYZ has area 80 cm^2.
Find the length of the equal sides.



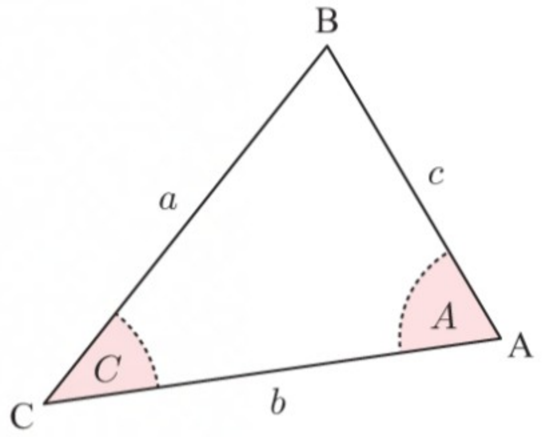
8 [AB] is the diameter of a circle with centre O and radius r.
Show that the shaded triangles have equal area.



9 **a** Find the area of triangle ABC using:

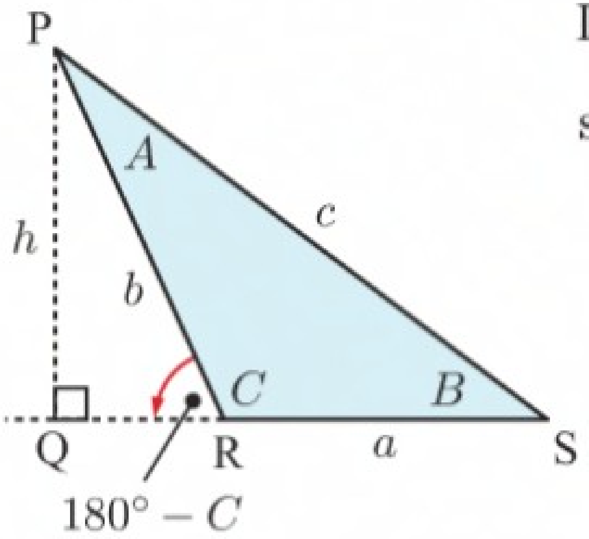
  i angle A **ii** angle C

b Hence, show that $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$.



1 **a** $\approx 55.2\text{ cm}^2$ **b** $\approx 347\text{ km}^2$ **c** $\approx 1.15\text{ m}^2$

2



In △PQR,

$$\sin(180^\circ - C) = \frac{h}{b}$$

$$\therefore \quad h = b\sin(180^\circ - C)$$

$$\therefore \quad h = b\sin C$$

But, area △PRS $= \frac{1}{2}ah$

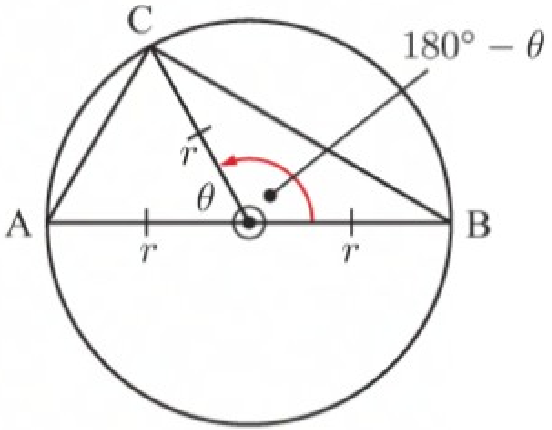
$$= \tfrac{1}{2}ab\sin C$$

3 **a** $\approx 13.6\text{ cm}^2$ **b** $\approx 58.6\text{ m}^2$ **c** $\approx 5.81\text{ m}^2$

4 **a** $\approx 41.6 \text{ cm}^2$ **b** $\approx 36.7 \text{ m}^2$ **c** $\approx 7.70 \text{ cm}^2$

5 50.0 cm^2 **6** $x \approx 21.9$ **7** ≈ 13.1 cm

8



Area △OBC

$= \frac{1}{2}r^2 \sin(180° - \theta)$

$= \frac{1}{2}r^2 \sin\theta$

$=$ area △OAC

9 **a** **i** Area $= \frac{1}{2}bc\sin A$ **ii** Area $= \frac{1}{2}ab\sin C$

b From **a**, $\frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C$

Divide both sides by $\frac{1}{2}abc$, etc.